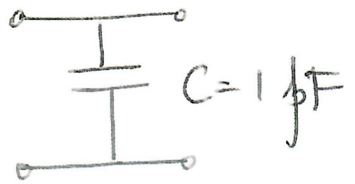
HOMEWORK ~~3~~ 2 SOLUTIONS

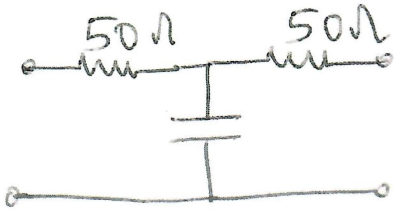
To find the Y_{an} and S matrix of the following ckt

Given $R_1 = R_2 = 50\Omega$ & $f = 10\,GHz$

a)



$$Z = \frac{1}{sC}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$



$$Z_a = R + Z = \begin{bmatrix} R + \frac{1}{sC} & \frac{1}{sC} \\ \frac{1}{sC} & R + \frac{1}{sC} \end{bmatrix}$$

$$\left[\because R = \begin{bmatrix} R & 0 \\ 0 & R \end{bmatrix} \right]$$

Let $x = 1/sC$

$$Y_a = \frac{1}{R^2 + 2Rx}\begin{bmatrix} x+R & -x \\ -x & x+R \end{bmatrix}$$

$$Y_{an} = \frac{1}{R+2x}\begin{bmatrix} x+R & -x \\ -x & x+R \end{bmatrix} = \frac{sC}{2 + sRC}\begin{bmatrix} \frac{1+RCs}{sC} & -\frac{1}{sC} \\ -\frac{1}{sC} & \frac{1+RCs}{sC} \end{bmatrix}$$

$$Y_{an} = \begin{bmatrix} \frac{1+3.14j}{2+3.14j} & -\frac{1}{2+3.14j} \\ -\frac{1}{2+3.14j} & \frac{1+3.14j}{2+3.14j} \end{bmatrix}$$

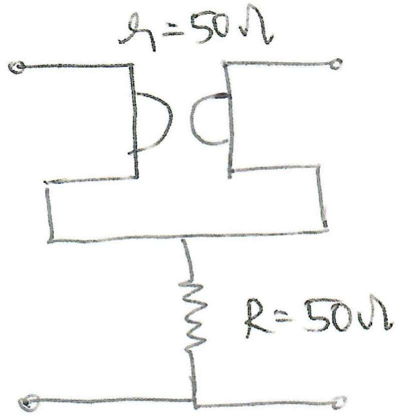
$$S = I - 2Y_{an} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} \frac{x+R}{2x+R} & -\frac{x}{R+2x} \\ -\frac{x}{R+2x} & \frac{x+R}{2x+R} \end{bmatrix}$$

$$\therefore S = \frac{1}{R + \frac{2}{j\omega C}} \begin{bmatrix} -R & \frac{2}{j\omega C} \\ \frac{2}{j\omega C} & -R \end{bmatrix}$$

For $R = 50\Omega$, $C = 1\,pF$ & $\omega = 2\pi \times 10 \times 10^{9}$ rad/s

$$\therefore S = \frac{-1}{50 - j31.83} \begin{bmatrix} 50 & j31.83 \\ j31.83 & 50 \end{bmatrix}$$

b)



$$Z_{gyrator} = \begin{bmatrix} 0 & -r \\ r & 0 \end{bmatrix} \qquad Z_R = \begin{bmatrix} R & R \\ R & R \end{bmatrix}$$

$$Z_{gyrator} + Z_R = \begin{bmatrix} R & R-r \\ R+r & R \end{bmatrix}$$

$$Z_a = Z_{gyrator} + Z_R + \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} = \begin{bmatrix} R+R_1 & R-r \\ R+r & R+R_2 \end{bmatrix}$$

$$Z_a = \begin{bmatrix} 100 & 0 \\ 100 & 100 \end{bmatrix} \Omega \qquad |Z_a| = (100)^2$$

$$Y_a = \frac{100}{(100)^2}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \frac{1}{100}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$Y_{an} = \frac{50}{100}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ -1/2 & 1/2 \end{bmatrix}$$

$$S = 1 - 2Y_{an} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$